

Probabilités continues

Terminale Option Mathématiques complémentaires

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Variable aléatoire à densité sur I

Fonction de densité sur I : fonction f continue et positive sur I telle que

$$\int_I f(t) dt = 1$$

$$\diamond P(a \leq X \leq b) = \int_a^b f(t) dt$$

$$\diamond P(X = a) = 0$$

$$\diamond P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$$

$$\diamond P(X \leq t) = 1 - P(X \geq t)$$

$$\diamond E(X) = \int_I t f(t) dt$$

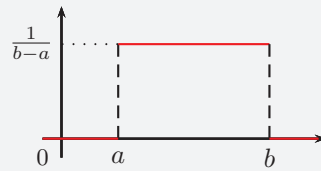
Loi uniforme sur $[a, b]$

Notation : $\mathcal{U}[a, b]$

$$f(t) = \frac{1}{b-a}$$

$$P(c \leq X \leq d) = \frac{d-c}{b-a}$$

$$E(X) = \frac{a+b}{2}$$



Loi exponentielle sur \mathbb{R}^+

Notation : $\mathcal{E}(\lambda)$

$$f(t) = \lambda e^{-\lambda t} \text{ avec } \lambda > 0$$

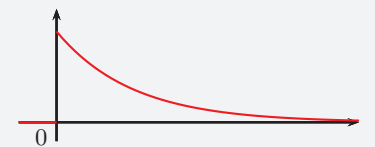
$$P(a \leq X \leq b) = e^{-\lambda a} - e^{-\lambda b}$$

$$P(X \leq t) = 1 - e^{-\lambda t}$$

$$P(X \geq t) = e^{-\lambda t}$$

$$P_{X \geq t}(X \geq t+h) = P(X \geq h)$$

$$E(X) = \frac{1}{\lambda}$$



Théorème de Moivre-Laplace

$$\diamond p \in]0, 1[\text{ et } n \in \mathbb{N}^*$$

$$\diamond X_n \text{ suit la loi binomiale } \mathcal{B}(n; p)$$

$$\diamond Z_n = \frac{X_n - np}{\sqrt{npq}}$$

Pour tous réels a et b tels que $a \leq b$:

$$\lim_{n \rightarrow +\infty} P(a \leq Z_n \leq b) = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

Loi normale centrée réduite sur \mathbb{R}

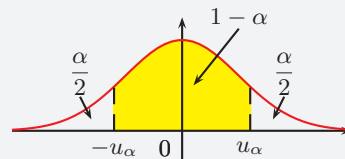
Notation : $\mathcal{N}(0; 1)$

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$$

$$E(X) = 0 \text{ et } V(X) = 1$$

$\forall \alpha \in]0, 1[$, $\exists ! u_\alpha \in \mathbb{R}_*^+$ tel que

$$P(-u_\alpha \leq X \leq u_\alpha) = 1 - \alpha$$



$$P(-1,96 \leq X \leq 1,96) = 0,95$$

$$P(-2,58 \leq X \leq 2,58) = 0,99$$

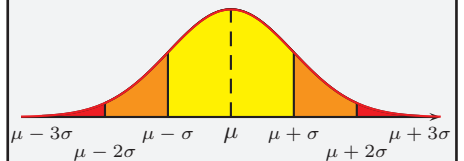
Loi normale sur \mathbb{R}

Notation : $\mathcal{N}(\mu; \sigma^2)$

$$E(X) = \mu \text{ et } V(X) = \sigma^2$$

X suit la loi $\mathcal{N}(\mu; \sigma^2) \iff$

$$Z = \frac{X - \mu}{\sigma} \text{ suit la loi } \mathcal{N}(0; 1)$$



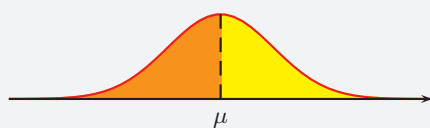
$$P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0,68$$

$$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 0,96$$

$$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 0,997$$

Propriétés des lois normales

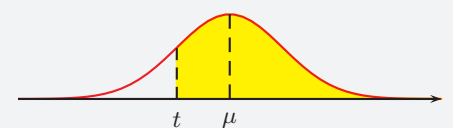
$$P(X \leq \mu) = P(X \geq \mu) = 0,5$$



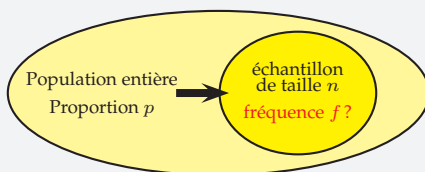
$$P(X < \mu - a) = P(X > \mu + a)$$



$$P(X > t) = 0,5 + P(t < X < \mu)$$



Intervalle de fluctuation



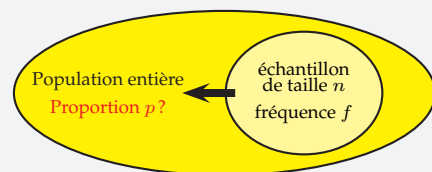
Intervalle de fluctuation asymptotique au seuil de $1 - \alpha$

$$I_n = \left[p - u_\alpha \sqrt{\frac{p(1-p)}{n}} ; p + u_\alpha \sqrt{\frac{p(1-p)}{n}} \right]$$

Conditions : $n \geq 30$; $np \geq 5$; $n(1-p) \geq 5$

à 95% : $u_{0,05} = 1,96$ et à 99% : $u_{0,01} = 2,58$

Intervalle de confiance



Intervalle de confiance de p au niveau de confiance 95%

$$\left[f - \frac{1}{\sqrt{n}} ; f + \frac{1}{\sqrt{n}} \right]$$

Conditions : $n \geq 30$; $nf \geq 5$; $n(1-f) \geq 5$