

# **REGULATIONS OF PUPILS' MISTAKES IN TEACHING: WHAT PURPOSES DO THEY SERVE?**

An Analysis of the Teaching Practice in 8 Classes during Lessons of Arithmetics

Marie-Pierre Chopin

DAESL, Laboratory LACES, Victor Segalen Bordeaux 2 University

“The advice given to teachers to make their professional practice more effective in mathematics teaching concerns essentially three points: the way of addressing pupils, the importance of synthesis and consideration of pupils' mistakes.”

IGEN, *L'enseignement des mathématiques au cycle 3 de l'école primaire*, 2006<sup>1</sup>

The pressure on teachers to regulate pupils' mistakes during lessons is as considerable in France as in any other country. In theory, its aim is an improvement of learning, especially in case of bad pupils. In practice, it can be time-consuming and does not always achieve the anticipated effects. The analysis presented here proposes an explanation of this phenomenon. It is based on the results of a research on arithmetical teaching with 197 10-11 year-old pupils (Chopin 2007). After presentation of the carried out experiment and after definition of methodology of making transcriptions and of studying teachers' regulations, we will study their function in didactic time progress: how these regulations are actually used during teaching. And above all, how to understand why teachers proceed in this way.

## **1. Presentation of the study**

The study is experimental. It is based on the comparison of lessons of 8 teachers in the last grade of elementary school.

### *a. About the knowledge*

All the lessons observed deal with the same mathematical domain, relational calculus, more exactly with problems about the fourth additive structure of

---

<sup>1</sup> This text comes from an official report of General Inspection of National Education (what we call IGEN in France) about mathematics teaching of 8 to 11 year-old pupils (i.e. the last two grades of elementary school in France). IGEN is an institution established in 1802. It is directly under the minister's authority. Its duty is evaluation and assessment of teaching, as well as making propositions about educational policy in France.

Vergnaud's typology (1989; 2009): TTT problems<sup>2</sup>. Here is for example a classical problem:

*Louise plays two games of marbles. She plays the first game. In the second one, she loses 4 marbles. After playing both games, she has won a total of 6 marbles. What happened in the first game?*

This kind of problem deals only with two operations: addition and subtraction. Furthermore, the operative difficulty is limited by the choice of numbers smaller than 10. This corresponds to the level of knowledge of 10-11 year-old pupils. On the other hand, these problems are characterized by the lack of information about the initial state: we never know how many marbles Louise had before playing the games. This difficulty is new in all of the observed classes. Moreover, when pupils are confronted with a pre-test based on 22 TTT problems of various difficulties, results show that the success of classes is comparable (the initial ability was similar in all the 8 classes)<sup>3</sup>.

#### *b. Different time for teaching*

The subject matter has been presented to teachers (individually). Subsequently they were asked to prepare lessons in such a way as to increase the success of their pupils in this domain. However, we did not give the 8 teachers the same amount of time to accomplish their lesson:

- Four were given 2 hours (2 lessons of 1 hour) ;
- Four were given 4 hours (4 lessons of 1 hour).

To put it shortly, some teachers (4h-classes) disposed with twice as much time as the others (2h-classes) to make the same teaching with pupils of equal ability. This device allows us to answer a few questions: will the regulations of mistakes be equally large in both groups of classes (2h and 4h)? Will they be as frequent? What about their nature? And above all, how do they affect pupils' comprehension?

#### *c. Measurement of pupils' progress*

At the end of the lessons, pupils were confronted with a post-test, similar to the pre-test, in order to measure their progress<sup>4</sup>. To secure validity, the pupils were assigned one more test a few weeks after the end of the lessons. It is called a « re-test » and permits to assess the durability of the acquired knowledge and skills 6 weeks after the end of the teaching.

---

<sup>2</sup> The specificity of this structure is that only positive or negative transformations are put in the game, without any indication of the initial numerical state – that is why it is called: " TTT " (" 1<sup>st</sup> Transformation – 2<sup>nd</sup> Transformation – composed Transformation ").

<sup>3</sup> An analysis of variation showed there was no significant difference among the 8 classes.

<sup>4</sup> This measurement is based on an index of progress built by Sarrazy (1996). It is too complex to be presented here. For more details, see Sarrazy (1996) or Chopin (2007).

## 2. Transcription of the teachers' regulations

The lessons of the 8 teachers were video recorded. It allowed us to transcribe the class discourse and to uncover in each of the classes the series of regulations in the lesson, that is all the teachers' interventions permitting them to direct the pupils, to correct their mistakes, etc. These regulations were recorded and analysed according two criteria: their nature (what is the content of the regulation?) and the time of their appearance in the lesson.

### *a. Nature of regulations*

Four types of regulation were distinguished in the teachers' practices:

- **Phatic regulations.** The teacher notices pupil's mistake but the regulation is rather superficial on the cognitive plan. Its aim is to preserve the didactic relationship with the pupil. For example the teacher uses the Topaze effect (Brousseau, 1987 ; Novotná & Hošpesová 2007), that is suggests the correct answer to the pupil by making his/her question more and more explicit (and that despite the fact that by doing that he/she modifies the knowledge expected for production of the correct answer);
- **Presentation or a reminder of a rule, a method, etc. connected with the relevant knowledge,** which is expected to help pupils overcome the mistake. The teacher is entirely responsible for the regulation, teaching directly or « showing » the knowledge necessary for production of the correct answer ;
- **Changes in the teaching device.** In the observed classes, the teachers sometimes modified their teaching device to regulate pupils' mistakes. But these modifications are often superficial, that is they do not deal with the structure of the problem pupils have to solve. The teacher only modifies the wording of the problems, he/she asks several pupils to work together in order to overcome their difficulties, etc.;
- **Organisation of a debate** with the whole class or a few pupils, where the teacher essentially takes the role of an activity leader. With this kind of regulation, the only instruction for pupils is to discuss the problem in order to find how to overcome their mistakes.

### *b. The point of regulation*

We have distinguished two instances in which regulations are employed by teachers:

- **Activity phases** where pupils work individually or in small groups to solve problems or exercises assigned by the teacher. Many mistakes produced by pupils occur at that time and the teacher who usually walks from a desk to a desk notices them;

- **The phases of collective correction** where all pupils and their teacher are engaged in the same kind of activity (presentation of answers, corrections, etc.).

On the basis of this schedule of analysis, we compared the regulations made by 2h-class teachers and 4h-class teachers. Thus we wanted to find stabilities and variations in how they are used.

### 3. Results

Four results will be presented here. They will be joined in an analysis in the fourth and last part of the text.

#### *a. Regulations are a tool used by teachers for the progress of didactic time (result 1)*

We have calculated the frequency of the regulations in each class (how many regulations on average in one hour of teaching?). The results show that the less time, the more frequent regulations are. The regulation frequency is on average more important in 2h-classes (9.9 regulations/h) than in 4h-classes (6.6 regulations/h). This means that regulations are considered by teachers as a real instrument for the progress of didactic time: they are a tool for teaching. The following result will complete this first assertion.

#### *b. The quantity of regulations is not linked with what pupils master in the lesson (result 2)*

If given more time, teachers have been able to make more regulations: we have counted an average of 26 regulations in 4h-classes, while only 19 in 2h-classes. This result is not surprising. What is surprising is that this difference has not lead to different pupils' progress in the two groups of classes. This is illustrated by the following graph:

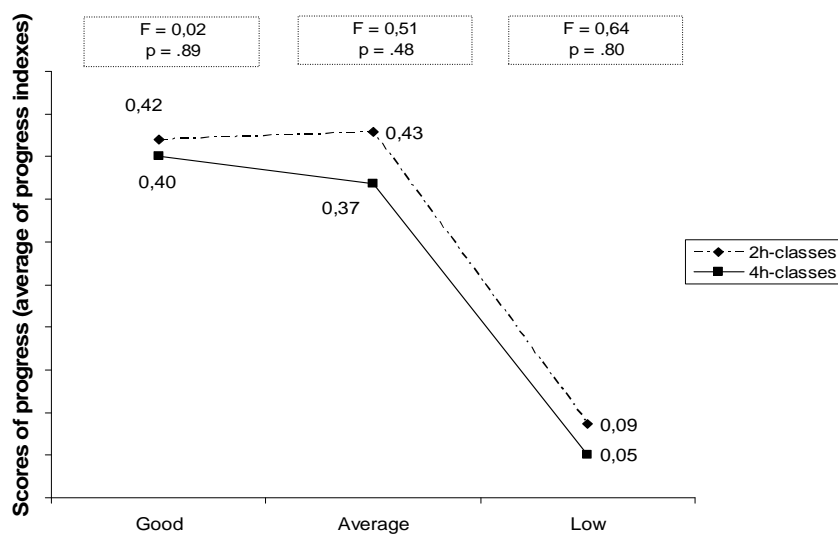


Figure 1 – Progress of pupils in 2h-classes and 4h-classes

The analysis of variance (rectangles at the top of Figure 1) shows that pupils' progress is the same whatever the school level is (good, average or low). Also the results of the re-test (6 weeks after the end of the lessons) permit to say that acquisitions are more sustainable in 2h-classes than in 4h-classes. These results may be surprising at the first sight. They will be explained later, with presentation of other results.

*c. The nature of regulations changes according to the time available for teaching (result 3)*

The nature of regulations in both groups of classes significantly changes in dependence on the time available for teaching ( $\chi^2 = 12,07$  ;  $p = .01$ ) :

<b>Content of the regulation</b>				
	Phatic regulations	Rule, method...	Change of device	Organization of a debate
<b>2h-CLA</b>	26	36	5	12
<b>4h-CLA</b>	21	36	17	31

Table 1 – Distribution of the types of regulation in 2h-classes and 4h-classes

Both in 2 hours and 4 hours, the teachers use phatic regulations and regulations based on teaching a rule or a method connected with the knowledge in the same quantity. Nevertheless, variation between the two groups may be observed in case of changing the teaching device and of organization of a debate: if given more time, these kinds of regulations are more important. Such a result underlines that, when conditions (here temporal conditions) permit it, teachers spend more time regulating pupils' mistakes (since they do not treat them in a phatic way) and that the regulation of these mistakes is not based on a directive teaching of the correct answer or method. On the contrary, teachers urge their pupils to overcome their difficulties by themselves, by confrontation with a new teaching device or by exchanging ideas with other pupils (organization of a debate).

*d. With more time, regulations occur more collectively than individually (result 4)*

The last result concerns the moment of regulation organization. In 2h-classes, 41% of regulations occur in the activity phase when pupils work alone or in small groups. In 4h-classes, only 26% of regulations occur during activity phases. In other words, the more time the teachers have, the more they regulate collectively (at the cost of individual regulations). Even if we have supposed that the important use of the debate as a regulation mode entails an increase in collective forms of regulations in 4h-classes (a debate means participation of the whole class), we could have thought that with more time the teachers would

have made more individual regulations during the activity phases (as we know that individual regulations are very time-consuming). This result will be explained in the fourth and last part.

#### **4. Interpretation and discussion**

In theory, regulation of pupils' mistakes aims at allowing them to overcome conceptual obstacles they meet in the lesson. This definition of regulation can be considered as "cognitive": it deals with the conceptual activity of pupils. Our results show that in practice, things are quite different.

In this study, teachers seem to give importance to regulation of pupils' mistakes. They want these regulations to be as effective as possible. The more time they have, the more they intent to develop a special pedagogy considering the domain of regulations: phatic regulations are less frequent and teachers try to have the pupils become actors of the regulation of their own mistakes, favouring device changes or collective debate instead of ostensive teaching. How can we understand that these practices do not trigger progresses? How to explain that 4h-class pupils do not make a bigger progress than 2h-class pupils?

One hypothesis could be that, beyond the superficial differences in the kinds of regulation in both groups of classes, the situations (in the way of the Theory of Didactic Situations, Brousseau 1987) organized by teachers to improve pupils' knowledge are not very different in 2h-classes and 4h-classes. In particular, their degree of adidacticity do not allow learning to appear without any explicit teaching (we have shown that elsewhere, cf. Chopin 2007). We have observed that in 4h-classes the regulations are predominantly organized in a collective way (result 4). Such moments could have been used by the teachers to make a kind of "proxy teaching": in debates, it is often the best pupils who teach the others what the teacher did not want to say (Brousseau 1987). Finally, and despite the superficial differences, the teachers' regulations play the same part in the progress of didactic time in both groups of classes.

This underlines the aspect necessarily didactic (and not strictly cognitive) of teachers' regulations: these regulations play an essential role in teaching the whole class; they do not directly aim at learning of each pupil individually. This idea is linked with the status of mistakes in teaching, as noticed by Raveinsten and Sensevy (1994, p.1):

“How disrupted could be the events in the classroom if the pupil, in the way of ideal teacher, stopped to make mistakes! What could be the substance, the first matter, from which could be built, with the teacher, the dialogue about learning? The mistake, by the discourse it involves, by the message it sends, by the marks it creates, is really the main vehicle of the communication about what makes the essential of the didactic relation [...].”

Still the mistake must keep its didactic dimension. Because the present tendency, which appears for example in France in the official reports on elementary teaching, is based on the idea that an efficient regulation of pupils' mistakes is necessarily cognitive and private. Here is the official report quoted at the beginning of our text, about mathematics teaching in France (IGEN 2006, p. 49):

“In many classes, mistakes are analyzed and deconstructed. But too often, the analysis of those mistakes occurs collectively. [...] If a single pupil writes 604 as result of  $600 + 40$ , it is not useful to analyse the mistake collectively. It is better to see discreetly if the pupil reproduces that error and, in that case, to look for the reason with him.”

This advice given to teachers gives evidence to the fact that as far as regulation of pupils' mistakes is concerned, didactical aspects might disappear behind cognitive ones. A mistake is considered as a personal feature of the pupil which must be regulated as such, that is personally. It is only when it concerns a “sufficient” number of pupils of the class, that it is to be regulated collectively. Such a perspective dangerously denies the far more complex status of the mistake (Salin (1976)) as well as the social dimension of learning. It overshadows that the mistake is linked with the conceptual stake of the lesson and that its appearance often says something important about the didactic play which involves the whole class.

We can therefore conclude that teachers are confronted with a real tension. In theory, they are asked to produce individual and private regulations; in practice, they find themselves confronted with strictly didactic necessities, little compatible with the cognitive and private definition of regulations. Contrary to an assumption, it is not the lack of time which prevents them from regulating the mistakes of all the pupils individually: these are functional necessities for the progress of didactic time. In consequence, mistakes are not harmful to the didactic time progress; on the contrary they are a condition for this progress. In the study presented here, calculus of correlations allowed to show that pupils made bigger progress if their mistakes were treated in a phatic way ( $p. < .05$ ) – especially in case of weak pupils! In the light of such results, the importance assigned nowadays to the issue of regulation of pupils' difficulties must be questioned. It seems that these can be favourable to progress only if they can keep their full didactic dimension, that is they remain an instrument of the teaching.

### **Bibliographical references**

Brousseau G. (1987). *Theory of Didactical situations in mathematics 1970-1990* [Translation from French M. Cooper, N. Balacheff, R. Sutherland & V. Warfield. Kluwer Academix Publishers, 304 p.

- Chopin M.-P. (2007). *Le temps didactique dans l'enseignement des mathématiques. Approche des modes de régulation des hétérogénéités didactiques*. Thèse pour le doctorat de l'Université Victor Segalen Bordeaux 2, 337 p.
- IGEN (2006). *L'enseignement des mathématiques au cycle 3 de l'école primaire*. Paris: Ministère de l'éducation nationale, de l'enseignement supérieur et de la recherche. Rapport n°2006-034, 70 p.
- Novotná J. & Hošpesová A. (2007). « What is the Price of Topaze ? » In Woo, J. H., Lew, H. C., Park, K. S. & Seo, D. Y. (Eds.). *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, 25-32. Seoul: PME.
- Salin M.-H. (1976). *Le rôle de l'erreur dans l'apprentissage des mathématiques à l'école primaire*. Mémoire de DEA de didactique des mathématiques, LADIST
- Sarrazy B. (1996). *La sensibilité au contrat didactique. Rôle des Arrières-plans dans la résolution de problèmes d'arithmétique au cycle trois*. Thèse pour le doctorat de l'Université de Bordeaux 2, 775 p.
- Vergnaud G (1990). « Développement et fonctionnement cognitifs dans le champ conceptuel des structures additives », In G. Netchine-Grynberg, *Développement et fonctionnement cognitifs chez l'enfant*. Paris: Presses Universitaires de France, 277 p.
- Vergnaud G. (2009). « The Theory of Conceptual Fields », *Human Development*, 52, 83-94.