Dirac field of negative energy and primordial antimatter incursion.

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Abstract : In this article we introduce Dirac field of negative energy and we show how this field is connected to Dirac field of positive energy. We also show that the electric charges of the negative energy field are exactly opposite to that of the positive energy field. Finally we show, within the framework of QFT, how we can transform the state of a fermion of positive (negative) mass and energy into a state of a fermion of negative (positive) mass and energy by using electromagnetic potential. This gives another possible explanation for the antiprotons excess observed by AMS-02 in 2016 as a primordial antimatter incursion.

Keywords : negative mass, negative energy, Dirac field, primordial antimatter, quantum field theory, mass inversion, dark matter.

1 Introduction

In order to solve the C asymmetry and baryons asymmetry, (i.e. the missing primordial antimatter), A. Sakharov in 1967 [1] introduced the idea of two universes with opposite time arrows and born at the initial singularity (i.e. big bang). In this way the CPT symmetry is preserved for both universes as a whole. In 2001 J-P. Petit et al.[2] have established that the reversal of time in one of Sakharov's universes must be accompanied by an inversion of mass and energy. To do this, they used Souriau's dynamical group theory[3]. In 2018, Debergh et al.[4] have shown that the exclusion of negative energies and masses in quantum mechanics through the use of an antiunitary and antilinear T (time reversal) operator is a simple hypothesis and that nothing allows to exclude the use of a unitary and linear T operator. They argue that there are two types of antimatter. The Dirac one created in the laboratory that is associated with an antiunitary PT transformation and the missing one or Feynman one that belongs to the twin universe of Sakharov, that is associated with the PT unitary transformation and which consists of the same particles but of negative mass and energy. Therefore, the negative masses of the Sakharov's twin universe as a substitute for the dark matter had to be considered and this is what Janus cosmological model (JCM) does [5, 6, 7]. For Sakharov, the two universes are related only by the initial singularity. In JCM it is also assumed that these universes interact but only by gravitation (i.e. antigravitation). So, negative matter is optically invisible or dark. JCM generalise Einstein theory of gravitation. It uses two metrics and by this way the "runaway" effect[8] is eliminated. The latter has always been mentioned as argument to reject the existence of negative mass even if Dirac theory does not prohibit such masses[9, 10]. There are other bimetric theories such as [11, 12, 13, 14, 15]. But the JCM fit with many observationals data[5, 6] and gives the first coherent description of the "Great Reppeller"[16].

Within the framework of QFT of Dirac taking into account a unitary time reversal operation we will introduce the Dirac field of negative energy and show that we can induce a transfer of a fermion from one universe to another. Actually, this is a physical inversion of mass. Quantum states of positive and negative energies (i.e. mass $+\mu$ and $-\mu$) can be coupled by using an electromagnetic potential. The last one allows transition from one state to another with a non zero probability. Since our goal is to show the feasibility of mass inversion we need to avoid the use of perturbative computations because mass inversion implies a quantum resonance effect which only manifests itself if the potential is relatively large compared to the energy of the relativistic particle. This is why we will limit ourselves to a uniform and time independant electromagnetic potential. In fact, such potential does indeed act on charged particle as shown by the Aharanov-Bohm effect [17, 18] in regions of space where electromagnetic field is zero.

The physical inversion of mass presented here is a first track that could give others to eventually provide the means to test in the laboratory the JCM. Indeed, according to JCM, if after inversion the mass +μ becomes -μ, the particle will be repelled by the earth instead of being attracted. Also, this mass inversion offers another possible explanation than dark matter annihilation for the excess of antiprotons observed by AMS-02 in 2016[19]. This excess could be the incursion of primordial antimatter (i.e. antiprotons) from the "twin universe" of Sakharov after mass inversion from -μ to +μ.

2 Dirac field in flat spacetime.

Dirac's field equation for free positive and negative mass in a covariant form is[20]:

$$\gamma^{\nu}\frac{\partial}{\partial x^{\nu}}\Psi_{\pm} \pm i\kappa\Psi_{\pm} = 0 \tag{1}$$

where $\kappa = c\mu/\hbar$. c and μ are respectively the speed of the light in vacuum and the mass at rest. Dirac fields operators with mass +μ and -μ are given respectively by:

$$\Psi_{+}(\mathbf{r}) = \int d^3\mathbf{p}\sum_{S}\Big(\Phi_{+,S,\mathbf{p}}(\mathbf{r})\mathrm{a}_{+,S,\mathbf{p}} + \ \Phi_{-,S,\mathbf{p}}(\mathbf{r})\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}\Big), \tag{2}$$

and

$$\Psi_{-}(\mathbf{r}) = \int d^3\mathbf{p}\sum_{S}\Big(\Theta_{-,S,\mathbf{p}}(\mathbf{r})\mathrm{b}_{-,S,\mathbf{p}} + \ \Theta_{+,S,\mathbf{p}}(\mathbf{r})\ \mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}\Big). \tag{3}$$

$\mathrm{a}_{j,S,\mathbf{p}}$ ($\mathrm{b}_{j,S,\mathbf{p}}$) and $\mathrm{a}^{\dagger}_{j,S,\mathbf{p}}$ ($\mathrm{b}^{\dagger}_{j,S,\mathbf{p}}$) are the operators of annihilation and creation of fermions of positive (negative) mass in the space of the number of occupations of the one-particle state characterized by quantum numbers j, S and $\mathbf{p}$. They obey the usual anticommutation relations:

$$\{\mathrm{a}_{j,S,\mathbf{p}}\ ,\ \mathrm{a}^{\dagger}_{j',S',\mathbf{p}'}\} = \delta_{j,j'}\ \delta_{S,S'}\delta(\mathbf{p}-\mathbf{p}') = \{\mathrm{b}_{j,S,\mathbf{p}}\ ,\ \mathrm{b}^{\dagger}_{j',S',\mathbf{p}'}\} \tag{4}$$

$$\{\mathrm{a}^{\dagger}_{j,S,\mathbf{p}}\ ,\ \mathrm{a}^{\dagger}_{j',S',\mathbf{p}'}\} = 0 = \{\mathrm{b}^{\dagger}_{j,S,\mathbf{p}}\ ,\ \mathrm{b}^{\dagger}_{j',S',\mathbf{p}'}\}, \tag{5}$$

$$\{\mathrm{a}_{j,S,\mathbf{p}}\ ,\ \mathrm{a}_{j',S',\mathbf{p}'}\} = 0 = \{\mathrm{b}_{j,S,\mathbf{p}}\ ,\ \mathrm{b}_{j',S',\mathbf{p}'}\}. \tag{6}$$

The index $j = (+, -)$ characterizes positive and negative frequencies. $S = (R, L)$ is the state of helicity. The vector $\mathbf{p}$ is the classical canonical variable of momentum conjugated to the vector of position $\mathbf{r}$. $\Phi_{j,S,\mathbf{p}}(\mathbf{r})$ and $\Theta_{j,S,\mathbf{p}}(\mathbf{r})$ are the stationary states of:

$$H_{\mathrm{o}}^{(+)}\ \ \Phi_{j,S,\mathbf{p}}(\mathbf{r}) = j\ \varepsilon(\mathbf{p})\ \ \Phi_{j,S,\mathbf{p}}(\mathbf{r})\ . \tag{7}$$

and:

$$H_{\mathrm{o}}^{(-)}\ \ \Theta_{j,S,\mathbf{p}}(\mathbf{r}) = j\ \varepsilon(\mathbf{p})\ \ \Theta_{j,S,\mathbf{p}}(\mathbf{r}) \tag{8}$$

with the eigenvalue of energy $\varepsilon(\mathbf{p})$ given by:

$$\varepsilon(\mathbf{p}) = (\mathrm{c}^2\mathbf{p}^2 + \mu^2\mathrm{c}^4)^{1/2}\ . \tag{9}$$

The one-particle hamiltonian operator $H_{\mathrm{o}}^{(\pm)}$ of Dirac for a free particle of spin 1/2 and mass $\pm\mu$ is given by:

$$H_{\mathrm{o}}^{(\pm)} = \mathrm{c}\alpha\cdot\mathbf{P} \pm \beta\mu\mathrm{c}^{\mathbf{2}}\ . \tag{10}$$

β and α_u (u = x, y, z) are the Dirac matrices:

$$\alpha_u = \begin{pmatrix} 0 & \sigma_u \\ \sigma_u & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix} \tag{11}$$

where σ_u are the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{12}$$

I is the matrix unit 2×2 and $\mathbf{P} \to -i\hbar\nabla$. Orthonormal wave functions can be expressed as follows:

$$\Phi_{j,S,\mathbf{p}}(\mathbf{r}) = \phi_{j,S,\mathbf{p}} \frac{\mathrm{e}^{i(\mathbf{p}\cdot\mathbf{r}/\hbar)}}{\sqrt{(2\pi\hbar)^3}} \qquad \Theta_{j,S,\mathbf{p}}(\mathbf{r}) = \theta_{j,S,\mathbf{p}} \frac{\mathrm{e}^{i(\mathbf{p}\cdot\mathbf{r}/\hbar)}}{\sqrt{(2\pi\hbar)^3}} \tag{13}$$

where $\phi_{j,S,\mathbf{p}}$ are the two-spinors[20]:

$$\phi_{+,R,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} (1+\mathrm{n_z}) \\ (\mathrm{n_x}+i\mathrm{n_y}) \\ \frac{\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{\mathrm{cp}(\mathrm{n_x}+i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \end{pmatrix}, \quad \phi_{+,L,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} -(\mathrm{n_x}-i\mathrm{n_y}) \\ (1+\mathrm{n_z}) \\ \frac{\mathrm{cp}(\mathrm{n_x}-i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{-\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \end{pmatrix} \tag{14}$$

$$\phi_{-,R,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} \frac{-\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{-\mathrm{cp}(\mathrm{n_x}+i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ (1+\mathrm{n_z}) \\ (\mathrm{n_x}+i\mathrm{n_y}) \end{pmatrix}, \quad \phi_{-,L,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} \frac{-\mathrm{cp}(\mathrm{n_x}-i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ -(\mathrm{n_x}-i\mathrm{n_y}) \\ (1+\mathrm{n_z}) \end{pmatrix} \tag{15}$$

and

$$\theta_{+,R,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} \frac{\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{\mathrm{cp}(\mathrm{n_x}+i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ (1+\mathrm{n_z}) \\ (\mathrm{n_x}+i\mathrm{n_y}) \end{pmatrix}, \quad \theta_{+,L,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} \frac{\mathrm{cp}(\mathrm{n_x}-i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{-\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ -(\mathrm{n_x}-i\mathrm{n_y}) \\ (1+\mathrm{n_z}) \end{pmatrix} \tag{16}$$

$$\theta_{-,R,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} (1+\mathrm{n_z}) \\ (\mathrm{n_x}+i\mathrm{n_y}) \\ \frac{-\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{-\mathrm{cp}(\mathrm{n_x}+i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \end{pmatrix}, \quad \theta_{-,L,\mathbf{p}} = \mathcal{B}(\mathbf{p}) \begin{pmatrix} -(\mathrm{n_x}-i\mathrm{n_y}) \\ (1+\mathrm{n_z}) \\ \frac{-\mathrm{cp}(\mathrm{n_x}-i\mathrm{n_y})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \\ \frac{\mathrm{cp}(1+\mathrm{n_z})}{[\varepsilon(\mathbf{p})+\mu\mathrm{c}^2]} \end{pmatrix}. \tag{17}$$

($\mathrm{n_x}$, $\mathrm{n_y}$, $\mathrm{n_z}$) are the three components of the unit vector $\mathbf{p}/\mathrm{p}$ with $\mathrm{p} \equiv |\mathbf{p}|$. The normalization constant $\mathcal{B}(\mathbf{p})$ is obtained by taking: $\phi^\dagger_{j,S,\mathbf{p}}\phi_{j,S,\mathbf{p}} = 1 = \theta^\dagger_{j,S,\mathbf{p}}\theta_{j,S,\mathbf{p}}$:

$$\mathcal{B}(\mathbf{p}) = \frac{1}{2}\sqrt{\frac{\varepsilon(\mathbf{p})+\mu\mathrm{c}^2}{\varepsilon(\mathbf{p})(1+\mathrm{n_z})}}. \tag{18}$$

Note that $\phi_{j,S,\mathbf{p}}$ with $\mu \to -\mu$ are also solutions of $H_{\mathrm{o}}^{(-)}$. But it is trivial solutions; no new quantum states. The $\theta_{j,S,\mathbf{p}}$ solutions are new quantum states.

The Hamiltonian of the Dirac field for free particles with a positive and negative mass are simply given by[20]:

$$\mathcal{H}_{\mathrm{o}}^{(+)} \equiv \int d^3\mathbf{r}\, \Psi_+^\dagger(\mathbf{r}) H_{\mathrm{o}}^{(+)} \Psi_+(\mathbf{r}) + \mathrm{C} = \int d^3\mathbf{p} \sum_{j,S} \varepsilon(\mathbf{p})\, \mathrm{a}^\dagger_{j,S,\mathbf{p}} \mathrm{a}_{j,S,\mathbf{p}} \tag{19}$$

$$\mathcal{H}_0^{(-)} \equiv \int d^3\mathbf{r}\, \Psi_-^\dagger(\mathbf{r}) H_0^{(-)} \Psi_-(\mathbf{r}) - \mathrm{C} \;=\; - \int d^3\mathbf{p} \sum_{j,S} \varepsilon(\mathbf{p})\; \mathrm{b}^\dagger_{j,S,\mathbf{p}} \mathrm{b}_{j,S,\mathbf{p}} \tag{20}$$

where:

$$\mathrm{C} \equiv 2\delta(0) \int d^3\mathbf{p}\; \varepsilon(\mathbf{p}) \;. \tag{21}$$

The constant C in (21) is infinite. It is a singularity. But, we can see that with the negative energy fields the sum of the energies eliminates the constant C. Thus, there is no singularity when we take into account negative energies. Equations (14) and (16) give the two-spinors corresponding to the eigenvalue $j\varepsilon(\mathbf{p})$ = $\varepsilon(\mathbf{p})$ (positive frequency) while (15) and (17) give those associated with the eigenvalue $j\varepsilon(\mathbf{p})$ = -$\varepsilon(\mathbf{p})$ (negative frequency). In addition we have:

$$\phi^\dagger_{j,S,\mathbf{p}} \phi_{j',S',\mathbf{p}} \;=\; \delta_{j,j'} \delta_{S,S'} \;\; = \theta^\dagger_{j,S,\mathbf{p}} \theta_{j',S',\mathbf{p}} \;. \tag{22}$$

It should be noted that the two-spinors in (14)-(15) and (16)-(17) have well-defined helicity (R and L). They are the eigenspinors of the one-particle helicity operator ($\Sigma\cdot\mathbf{p}$)/p with eigenvalues $+1$ ($S = R$) and -1 ($S = L$):

$$\frac{1}{\mathrm{p}}(\Sigma\cdot\mathbf{p})\phi_{j,S,\mathbf{p}} = \pm\phi_{j,S,\mathbf{p}} \qquad \frac{1}{\mathrm{p}}(\Sigma\cdot\mathbf{p})\theta_{j,S,\mathbf{p}} = \pm\theta_{j,S,\mathbf{p}} \tag{23}$$

where Σ, in the standard representation, is given by:

$$\Sigma_u = \begin{pmatrix} \sigma_u & 0 \\ 0 & \sigma_u \end{pmatrix} \;. \tag{24}$$

Finally we must note that we have two vacua $|0\rangle_\mathrm{a}$ and $|0\rangle_\mathrm{b}$ defined by:

$$\mathrm{a}_{j,S,\mathbf{p}}|0\rangle_\mathrm{a} = 0 \qquad \mathrm{b}_{j,S,\mathbf{p}}|0\rangle_\mathrm{b} = 0 \;. \tag{25}$$

3 The meaning of negative energy

3.1 Unitary time reversal

Negative energy can be see or the result of unitary time-reversal. According to the work of Debergh et al.[4] the two-spinors of negative mass in (16) to (17) can be obtained from the two-spinors of positive mass (14) to (15) when we apply on these a unitary time reversal operation such as:

$$\begin{aligned} x &\to x \\ t &\to -t \\ \mathrm{i} &\to \mathrm{i} \\ E &\to -E \\ \mu &\to -\mu \\ \mathbf{p} &\to \mathbf{p} \;. \end{aligned} \tag{26}$$

By changing t to $-t$ in the energy operator, the energy changes sign since the imaginary number “i” does not change (unitary time reversal):

$$E \leftrightarrow i\hbar\frac{\partial}{\partial t} \to -i\hbar\frac{\partial}{\partial t} \leftrightarrow -E \;. \tag{27}$$

And since the mass μ is energy at rest (E = μc^2) we must also have a change of sign of the mass during a unitary time reversal. Note that this reversal leaves invariants exponentials:

$$\mathrm{e}^{-\mathrm{i}\frac{\varepsilon(\mathbf{p})\mathbf{t}}{\hbar}} \to \mathrm{e}^{-\mathrm{i}\frac{\varepsilon(\mathbf{p})\mathbf{t}}{\hbar}} \quad \text{and} \quad \mathrm{e}^{\mathrm{i}\frac{\mathbf{p}\cdot\mathbf{r}}{\hbar}} \to \mathrm{e}^{\mathrm{i}\frac{\mathbf{p}\cdot\mathbf{r}}{\hbar}} \;. \tag{28}$$

So, by doing $\varepsilon(\mathbf{p}) \to -\varepsilon(\mathbf{p})$ and $\mu \to -\mu$ in (14) to (15) we find equations (16) to (17) as:

$$\mathrm{T-unitary:} \qquad \phi_{\pm,S,\mathbf{p}} \to \theta_{\mp,S,\mathbf{p}} \ . \tag{29}$$

For comparison, an anti-unitary time reversal operation leads to:

$$\begin{aligned} x &\to x \\ t &\to -t \\ \mathrm{i} &\to -\mathrm{i} \\ E &\to E \\ \mu &\to \mu \\ \mathbf{p} &\to -\mathbf{p} \ . \end{aligned} \tag{30}$$

The last relation is the result of the operator:

$$\mathbf{P} \leftrightarrow -i\hbar\nabla \ . \tag{31}$$

3.2 Opposite phase factor.

Negative energy can also be seen as the result of taking an opposite phase factor of a quantum state. To see it, let's rewrite the two-spinors $\phi_{j,S,\mathbf{p}}$ and $\theta_{j,S,\mathbf{p}}$. The basis of representation used above for the two-spinors is the vector product of two basis of states and is of dimension 4: $|\eta\rangle_1 \otimes |\eta'\rangle_2 \equiv |\eta\rangle_1 |\eta'\rangle_2$ and they are given by:

$$|+\rangle_1|+\rangle_2 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |+\rangle_1|-\rangle_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |-\rangle_1|+\rangle_2 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |-\rangle_1|-\rangle_2 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{32}$$

These equalities come from the following prescription:

$$\begin{pmatrix}a\\b\end{pmatrix} \otimes \begin{pmatrix}\alpha\\\beta\end{pmatrix} = \begin{pmatrix}a\alpha\\a\beta\\b\alpha\\b\beta\end{pmatrix} \tag{33}$$

which gives a two-spinors from the vector product of two spinors and definitions:

$$|+\rangle_{\mathrm{k}} \equiv \begin{pmatrix}1\\0\end{pmatrix} \tag{34}$$

$$|-\rangle_{\mathrm{k}} \equiv \begin{pmatrix}0\\1\end{pmatrix} \tag{35}$$

where k = 1,2. Let's define two pairs of spinors states:

$$\begin{aligned} |+\rangle_{\mathrm{II}} &= \cos(\theta/2)\,|+\rangle_2 + \sin(\theta/2)e^{i\phi}\,|-\rangle_2 \\ |-\rangle_{\mathrm{II}} &= -\sin(\theta/2)e^{-i\phi}\,|+\rangle_2 + \cos(\theta/2)\,|-\rangle_2 \end{aligned} \tag{36}$$

which is the spin or helicity along momentum $\mathbf{p}$ and

$$\begin{aligned} |+\rangle_{\mathrm{I}} &= \cos(\varphi/2)\,|+\rangle_1 + \sin(\varphi/2)e^{i\chi}\,|-\rangle_1 \\ |-\rangle_{\mathrm{I}} &= -\sin(\varphi/2)e^{-i\chi}\,|+\rangle_1 + \cos(\varphi/2)\,|-\rangle_1\,. \end{aligned} \tag{37}$$

Taking into account the definitions, $\mathbf{p} = (p_x, p_y, p_z)$ and $n_u = p_u/|\mathbf{p}|$, we set:

$$\begin{aligned} n_x &= \sin(\theta)\cos(\phi) \\ n_y &= \sin(\theta)\sin(\phi) \\ n_z &= \cos(\theta) \end{aligned} \tag{38}$$

where $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. Parameter φ is defined by the relations:

$$\begin{aligned} \cos(\varphi) &= \frac{\mu c^2}{\varepsilon(\mathbf{p})} \\ \sin(\varphi) &= \frac{c|\mathbf{p}|}{\varepsilon(\mathbf{p})}\,, \end{aligned} \tag{39}$$

with $0 \leq \varphi \leq \pi/2$ and where $0 \leq \chi \leq 2\pi$. In accordance with the definitions given above, the equations (14)-(15) and (16)-(17) take simple forms:

$$\phi_{+,R,\mathbf{p}} = |+\rangle_{\mathrm{I}(\chi=0)}\,|+\rangle_{\mathrm{II}} \qquad \theta_{-,R,\mathbf{p}} = |+\rangle_{\mathrm{I}(\chi=\pi)}\,|+\rangle_{\mathrm{II}} \tag{40}$$

$$\phi_{+,L,\mathbf{p}} = |+\rangle_{\mathrm{I}(\chi=\pi)}\,|-\rangle_{\mathrm{II}} \qquad \theta_{-,L,\mathbf{p}} = |+\rangle_{\mathrm{I}(\chi=0)}\,|-\rangle_{\mathrm{II}} \tag{41}$$

$$\phi_{-,R,\mathbf{p}} = |-\rangle_{\mathrm{I}(\chi=0)}|+\rangle_{\mathrm{II}} \qquad \theta_{+,R,\mathbf{p}} = |-\rangle_{\mathrm{I}(\chi=\pi)}\,|+\rangle_{\mathrm{II}} \tag{42}$$

$$\phi_{-,L,\mathbf{p}} = |-\rangle_{\mathrm{I}(\chi=\pi)}|-\rangle_{\mathrm{II}} \qquad \theta_{+,L,\mathbf{p}} = |-\rangle_{\mathrm{I}(\chi=0)}\,|-\rangle_{\mathrm{II}}. \tag{43}$$

According to (40)-(43), the phase factor χ of the states $\theta_{-j,S,\mathbf{p}}$ has opposite value (i.e. $\pm\pi$) relatevely to the $\phi_{j,S,\mathbf{p}}$ ones. This is equivalent to reverse the sign of mass and energy in the $\phi_{j,S,\mathbf{p}}$ or take a unitary time reversal.

4 Connection between fermions of mass +μ and -μ.

From (14) to (17) it can be show that:

$$\begin{pmatrix} \theta_{+,S,\mathbf{p}} \\ \theta_{-,S,\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \eta\sin(\varphi) & \cos(\varphi) \\ \cos(\varphi) & -\eta\sin(\varphi) \end{pmatrix} \begin{pmatrix} \phi_{+,S,\mathbf{p}} \\ \phi_{-,S,\mathbf{p}} \end{pmatrix} \tag{44}$$

with $\eta = +1$ for $S = R$ and -1 for $S = L$. So, there is a relation between the two kinds of two-spinors which is quite similar to a rotation in a plane. But the direction of rotation changes with the helicity. Because of these relations, there are similar relationships between operators $a_{j,S,\mathbf{p}}$ and $b_{j,S,\mathbf{p}}$. If we put (44) in equation (3) we can show by identification with (2) that:

$$\begin{pmatrix} a^\dagger_{-,S,-\mathbf{p}} \\ a_{+,S,\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \cos(\varphi) & -\eta\sin(\varphi) \\ \eta\sin(\varphi) & \cos(\varphi) \end{pmatrix} \begin{pmatrix} b^\dagger_{+,S,-\mathbf{p}} \\ b_{-,S,\mathbf{p}} \end{pmatrix} \tag{45}$$

and then $\Psi_{+}(\mathbf{r}) = \Psi_{-}(\mathbf{r}) \equiv \Psi(\mathbf{r})$. Equation (45) is a Bogoliubov transformation.

On the other hand, the charge operator Q is defined by[20]:

$$Q \equiv -\frac{e}{2} \int d^3\mathbf{r} \Big(\Psi^{\dagger}(\mathbf{r})\Psi(\mathbf{r}) - \widetilde{\Psi}(\mathbf{r})\widetilde{\Psi}^{\dagger}(\mathbf{r}) \Big) \tag{46}$$

where "$e > 0$" is the elementary electric charge and symbol "$\sim$" indicate matrix transposition. Therefore we get from (2)-(3) two representations of Q:

$$Q = -e \int d^3\mathbf{p} \sum_{j,S} \left(j\, a^{\dagger}_{j,S,\mathbf{p}} a_{j,S,\mathbf{p}} \right) = e \int d^3\mathbf{p} \sum_{j,S} \left(j\, b^{\dagger}_{j,S,\mathbf{p}} b_{j,S,\mathbf{p}} \right) . \tag{47}$$

As we can see in the expression of Q in terms of $a_{j,S,\mathbf{p}}$, we have fermions of negative charge (i.e. $j = +$) and fermions of positive charge (i.e. $j = -$). But, in terms of $b_{j,S,\mathbf{p}}$, we have fermions of positive charge (i.e. $j = +$) and fermion of negative charge (i.e. $j = -$). The signs of the charges are exactly opposite for a j given. So, index j (i.e. frequency) is also a quantum number for charge.

5 Mass inversion.

The expression "physical mass inversion" is an abuse of language. It does not reverse the mass by acting "directly" on it. We are acting with electromagnetic potential on a quantum state where the mass is +μ so that there is a non-negligible probability that it passes to another quantum state where the mass is -μ. The opposite is also true.

5.1 Coupling to electromagnetic potential.

It is well known [20] that when an electromagnetic potential A_ν imposed from the outside by a macroscopic source acts on an electric charge particle q, of mass $\pm\mu$ and of spin 1/2, equation (1) becomes[20]:

$$\gamma^{\nu} \left(\frac{\partial}{\partial x^{\nu}} + \frac{iq}{\hbar} A_{\nu} \right) \Psi_{\pm} \pm i\kappa \Psi_{\pm} = 0 \, . \tag{48}$$

In order to obtain numerical results it is more convenient to use the noncovariant form:

$$i\hbar \frac{\partial}{\partial t} \Psi_{\pm}(\mathbf{r},t) = H^{(\pm)} \Psi_{\pm}(\mathbf{r},t) \, . \tag{49}$$

$H^{(\pm)}$ is the Dirac hamiltonian operator given by:

$$H^{(\pm)} = c\alpha \cdot (\mathbf{P} - q\mathbf{A}) + q\Phi \pm \beta\mu c^2 \tag{50}$$

The magnetic potential $\mathbf{A}$ can couple states with differents index "j" because of matrix α and this is necessary to induce transitions among states of masses positive and negative. The scalar potential Φ cannot do that so it will be forgotten in the following without loss of generality.

In order to simplify the calculations, it is assumed that $\mathbf{A}$ is uniform (so, $\nabla \times \mathbf{A} = 0$) where is the particle and we admit that its direction in space is parallel to the linear momentum vector $\mathbf{p}$ of the particle so that $\mathbf{A} = A\mathbf{n}$ where $\mathbf{n} = \frac{\mathbf{p}}{|\mathbf{p}|}$ and A is a scalar independant of time.

Suppose that potential $\mathbf{A}$ is applied only to particles having a mass $+\mu$ and a charge q. So, only $H^{(+)}$ in (50) depends on $\mathbf{A}$ and $H^{(-)} = H_o^{(-)}$. As a result, only the field Ψ_{+} in (2) will be modified:

$$\Psi_{+}(\mathbf{r},t) = \int d^3\mathbf{p} \sum_{S} \Big(\Phi_{+,S,\mathbf{p}}(\mathbf{r}) a_{+,S,\mathbf{p}}(t) + \Phi_{-,S,\mathbf{p}}(\mathbf{r})\, a^{\dagger}_{-,S,-\mathbf{p}}(t) \Big), \tag{51}$$

Introducing (51) in (49) we find the equations of the movement of the operators “a”:

$$i\hbar\frac{\partial}{\partial t}\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) = \omega_o \mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) + \omega_1 \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) \; , \tag{52}$$

$$i\hbar\frac{\partial}{\partial t}\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) = -\omega_o \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) + \omega_1 \mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) \; . \tag{53}$$

Solutions are:

$$\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) = \mathrm{f}^*(\mathrm{t})\ \mathrm{a}_{+,S,\mathbf{p}} + \mathrm{g}^*(\mathrm{t})\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}} \tag{54}$$

$$\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) = \mathrm{f}(\mathrm{t})\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}} - \mathrm{g}(\mathrm{t})\ \mathrm{a}_{+,S,\mathbf{p}} \tag{55}$$

where:

$$\mathrm{f}(\mathrm{t}) = \mathrm{e}^{-\mathrm{i}\lambda \mathrm{t}/\hbar} + 2\mathrm{i}\Big(\frac{\omega_o+\lambda}{2\lambda}\Big)\sin(\lambda \mathrm{t}/\hbar) \tag{56}$$

$$\mathrm{g}(\mathrm{t}) = \mathrm{i}\Big(\frac{\omega_1}{\lambda}\Big)\sin(\lambda \mathrm{t}/\hbar) \tag{57}$$

with the initial conditions: f(0) = 1 and g(0) = 0 and:

$$\omega_o \equiv \varepsilon(\mathbf{p}) - \mathrm{c}qA\sin(\varphi) \qquad \omega_1 \equiv -\mathrm{c}qA\eta\cos(\varphi) \; , \tag{58}$$

$$\lambda = \sqrt{\omega_o^2+\omega_1^2} = \sqrt{(\mathrm{c}(\mathbf{p}-qA\mathbf{n}))^2+(\mu \mathrm{c}^2)^2} \; . \tag{59}$$

5.2 Probability of mass inversion.

As an example, consider the case of a particle of mass $+\mu$ and electric charge $q =$ -e in the vacuum $|0\rangle_{\mathrm{a}}$. The probability that having created this particle in the state (S, $\mathbf{p}$) at t = 0 we can recover it in the same state at t $>$ 0 is:

$$|{}_{\mathrm{a}}\langle 0|\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(0)|0\rangle_{\mathrm{a}}|^2 = \mathrm{f}^*(\mathrm{t})\mathrm{f}(\mathrm{t}) = 1 - G(\mathrm{t}) \; , \tag{60}$$

where:

$$G(\mathrm{t}) \equiv \Big(\frac{\omega_1}{\lambda}\Big)^2 \sin^2(\lambda \mathrm{t}/\hbar) \; . \tag{61}$$

When $A = 0$ this probability is always equal to 1. But when $A \neq 0$ the probability can become null. Indeed, there is a quantum resonance which appears when $(\frac{\omega_1}{\lambda})^2 = 1$ in (61). This happens if $\omega_o = 0$ which implies from (58):

$$qA = |\mathbf{p}|\Big[1+\Big(\frac{\mu \mathrm{c}^2}{\mathrm{c}|\mathbf{p}|}\Big)^2\Big] \; . \tag{62}$$

For $q =$ -e, we must have $A < 0$ and so $\mathbf{A}$ is opposed to $\mathbf{p}$. Therefore, at specific times and this periodically it is not possible to recover the particle because $G(\mathrm{t}) = 1$. Suppose t = $\mathrm{t_o}$ is one of those specific times. What is the probability at this moment $\mathrm{t_o}$ that the particle reverses its mass? Using (45) and (54) one can show that:

$$\begin{aligned} |{}_{\mathrm{b}}\langle 0|\mathrm{b}_{j,S',\mathbf{p}'}(\mathrm{t})\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(\mathrm{t})|0\rangle_{\mathrm{b}}|^2 &= \Big(1-G(\mathrm{t})\Big)\cos^2(\varphi) + G(\mathrm{t})\sin^2(\varphi) \\ &\quad -2\eta\Big(\frac{\omega_0}{\omega_1}\Big)G(\mathrm{t})\sin(\varphi)\cos(\varphi) \end{aligned} \tag{63}$$

only if $j = -$ (i.e. negative charge for “b” operators, see eq.(47)), $\mathbf{p}' = \mathbf{p}$ and $S' = S$ otherwise it’s zero and:

$$\mathrm{b}_{j,S,\mathbf{p}}(\mathrm{t}) = \mathrm{e}^{-\mathrm{i}\big(j\varepsilon(\mathbf{p})\mathrm{t}/\hbar\big)}\ \mathrm{b}_{j,S,\mathbf{p}} \; . \tag{64}$$

Equation (63) gives the probability that having created a particle of mass $+\mu$ and electric charge -e in a state (S, $\mathbf{p}$) at time t in vacuum $|0\rangle_{\mathrm{b}}$ we can recover it at the same moment in the same state with the same electric charge but of mass $-\mu$. Note that mass inversion does not reverse the electric charge. Then at t = t_o, $G(t_o) = 1$ and the probability is:

$$|_b\langle 0|b_{j,S',\mathbf{p}'}(t_o)a^{\dagger}_{+,S,\mathbf{p}}(t_o)|0\rangle_b|^2 = \sin^2(\varphi) = \left(\frac{c|\mathbf{p}|}{\varepsilon(\mathbf{p})}\right)^2 . \quad (65)$$

The more the particle is relativistic the more the probability approaches 1.

What we have done with a particle of mass $+\mu$ (i.e. $+\mu \to -\mu$) and electric charge -e we can do it again with a mass $-\mu$ (i.e. $-\mu \to +\mu$) and electric charge -e (or +e) with a potential applied only on states of negative masses. The results will be similar. This last situation corresponds to the case of primordial antiparticles which under the action of an electromagnetic potential would be "transferred" from the "twin universe" of Sakharov to ours.

Note that in 2016, the AMS-02 particle detector installed on the ISS measured an antiproton excess. The energy of the antiprotons was between 10 to 20 GeV. Cholis et al.[19] have considered the possibility that this excess is caused by the annihilation of massive particles of dark matter (48-67 GeV) consistent with a gamma excess of several GeV observed by Fermi-LAT in the center of the galaxy in 2011. After annihilation of massive particles there would be quark-antiquark production and finally antiprotons formation. However, it is interesting to note that the energy of these antiprotons is large enough so that the probability of mass inversion in (65) is very close to 1. In this case the value of potential A would be in the range of 30 to 60 Tesla·m.

6 Conclusion

In this work we have obtained, within the framework of the QFT of Dirac, the fields of negative energies for fermions of spin 1/2. We also show that there is a connection between positive and negative energy fields and that the electrical charges associated with these fields are opposite. As a result of applying an electromagnetic potential, we have shown how one could transfer a fermion of charge q, helicity S and linear momentum $\mathbf{p}$ in a mass state $+\mu$, to a fermion of same charge, helicity and linear momentum in a mass state $-\mu$. Such a transfer is of course possible in the other direction if an electromagnetic potential is applied on the electric charge of a particle of mass $-\mu$.

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